TD8 - First order logic IV

April 11, 2025

1. [*Homework 22/23*] Translate as precisely as possible the following sentences into predicate logic. In case of ambiguity, provide a formula for each possible reading.

   (a) All architects have built a bridge.

   (b) Sam is depressed when no one understands her.

   (c) Any company that disappoints some customers will disappear if they all complain.

   (d) Either no student will find an answer, or all of them will.

   (e) A doctor will be sent to anyone who rejects someone who loves them.

2. [*Homework 23/24*] In natural language (and also in science), discourse often has changing domains. Therefore, it is interesting to study what happens to the truth of formulae in a model when the model undergoes some transformation. Let us call *persistent* a formula whose truth is not affected by enlarging the models with new objects. Which of the following formulae are generally persistent?

   (1) a. $\exists x\ Px$
   b. $\forall x\ Px$
   c. $\exists x \forall y\ Rxy$
   d. $\neg \forall x \forall y\ Rxy$

3. Consider a model $\mathcal{M} = \langle \mathbb{N}, I \rangle$. Let p be a binary predicate such that $I(p) = \{(x, y) \mid y = 2x\}$. Among the following formulae, which are satisfied by $\mathcal{M}$?

   (a) $p\ 0\ 0$

   (b) $p\ 1\ 1$

   (c) $\exists x\ p\ x\ 1$

   (d) $\exists x\ p\ 1\ x$

   (e) $\forall x\ p\ 1\ x$

   (f) $\exists x \exists y\ p\ x\ y$

   (g) $\exists x \forall y\ p\ x\ y$

   (h) $\forall x \exists y\ p\ x\ y$

   (i) $\forall x \forall y\ p\ x\ y$

4. Consider two formulae $F_1 = \forall x \exists y\ p\ x\ y$ and $F_2 = \exists y \forall x\ p\ x\ y$.

   (a) Define a model $\mathcal{M}_1$ such that $[\![F_1]\!]^g_{\mathcal{M}_1} = [\![F_2]\!]^g_{\mathcal{M}_1} = 1$ for all assignments g.

   (b) Define a model $\mathcal{M}_2$ such that $[\![F_1]\!]^g_{\mathcal{M}_2} = [\![F_2]\!]^g_{\mathcal{M}_2} = 0$ for all assignments g.

   (c) Define a model $\mathcal{M}_3$ such that $[\![F_1]\!]^g_{\mathcal{M}_3} = 1$ and $[\![F_2]\!]^g_{\mathcal{M}_3} = 0$ for all assignments g.

   (d) There exists a model $\mathcal{M}_4$ such that $[\![F_1]\!]^g_{\mathcal{M}_4} = 0$ and $[\![F_2]\!]^g_{\mathcal{M}_4} = 1$ for all assignments g?