TD5 - First order logic I

March 21, 2025

1. Among the following expressions, which are well-formed formulae of L_p?

(1)	$\neg(\neg P \vee Q)$	(5)	$(P \to ((P \to Q)))$	(9)	$(P \vee (Q \vee R))$
(2)	$P \vee (Q)$	(6)	$((P \to P) \to (Q \to Q))$	(10)	$-P \vee Q \vee R$
(3)	$\neg(Q)$	(7)	$((P_{28} \to P_3) \to P_4)$	(11)	$(\neg P \vee \neg\neg P)$
(4)	$(P_2 \to (P_2 \to (P_2 \to P_2)))$	(8)	$(P \to (P \to Q) \to Q)$	(12)	$(P \vee P)$

2. Show that the connectives $\wedge$ and $\neg$ are sufficient, *i.e.* any formula containing other connectives ($\vee, \to, \leftrightarrow$) is equivalent to a formula having only $\wedge$ and $\neg$.

3. Alice, Bob and Charlie are suspected of tax fraud. They provide the following testimonies:

Alice:	Bob is guilty and Charlie is innocent.
Bob:	If Alice is guilty, Charlie is guilty too.
Charlie:	I am innocent but at least one of the other two is guilty.

   Express each suspect's testimony in propositional logic. Answer the following questions, using a truth table:

   (a) Is it possible for all three testimonies to be true simultaneously?

   (b) If all suspects are innocent, is someone lying, and if so, who?

   (c) If the guilty ones lie and the innocent ones tell the truth, who is innocent and who is guilty?

4. For each of the following predicate calculus formulas, indicate the scope of quantifiers and the free variables and specify whether it is a sentence.

   (a) $\exists x\ (Axy \wedge Bx)$
   (b) $\exists x\ Axy \wedge Bx$
   (c) $\exists x \exists y\ Axy \to Bx$
   (d) $\exists x\ (\exists y\ Axy \to Bx)$
   (e) $\neg\exists x \exists y\ Axy \to Bx$
   (f) $\forall x \neg\exists y\ Axy$
   (g) $\neg Bx \to (\neg\forall y\ (\neg Axy \vee Bx) \to Cy)$

   (a) $\exists x\ (Axy \vee By)$
   (b) $\exists x\ Axx \vee \exists y\ By$
   (c) $\exists x\ (\exists y\ Axy \vee By)$
   (d) $\forall x \forall y\ ((Axy \wedge By) \to \exists w\ Cxw)$
   (e) $\forall x\ (\forall y\ Ayx \to By)$
   (f) $\forall x \forall y\ Ayy \to Bx$