

TD2 - Regular languages

February 14, 2025

1. Propose complete deterministic finite automata that recognise the following languages:
 - (a) [*Exam 23/24*] The set of words that are composed of an even number of a 's, followed by an odd number of b 's ($\Sigma = \{a, b\}$).
 - (b) [*Homework 23/24*] The set of words on Σ^* such that all c 's are before all b 's (if any), the number of c 's is odd (thus > 1) and the number of a 's is even, and b 's can occur only if they are not followed by a 's ($\Sigma = \{a, b, c\}$).
2. Let $\Sigma = \{a, b\}$. Let $L_1 = \{a, ab, ba\}$ and $L_2 = \{\varepsilon, b, ab\}$ be two languages.
 - (a) Compute the following operations:

$$L_1.L_2 \quad L_2.L_1 \quad L_2 \setminus L_1 \quad L_1.\{\varepsilon\} \quad \emptyset.L_2$$
 - (b) If $L_3.L_4 = \{\varepsilon\}$, what can be said about languages L_3 and L_4 ?
 - (c) If $L_3.L_4 = \emptyset$, what can be said about languages L_3 and L_4 ?
3. Let $\Sigma = \{a, b\}$. Describe the following regular expressions in plain English and propose deterministic finite state automata that recognise the underlying languages:
 - (a) $(a|b)^*$
 - (b) $a(a|b)^*$
 - (c) $(b|ab)^*(a|\varepsilon)$
 - (d) $a^*|b^*$
 - (e) $(aa|b)^*$
 - (f) $(ab^*a|b)^*$
 - (g) $(b|ab|aab)^*a^3(a|b)^*$
4. Let $\Sigma = \{a, b, c\}$. Give regular expressions that accept the following languages:
 - (a) The set of words containing at least one letter.
 - (b) The set of words not ending with a b .
 - (c) The set of words with at least a b .
 - (d) The set of words containing at most a b .
5. Are the following languages rational? Justify.
 - (a) $L_1 = \{a^n b^n \mid n \in \mathbb{N}\}$
 - (b) $L_2 = \{a^m b^n \mid n \equiv m \pmod{3}\}$
 - (c) $L_3 = \{w \in \{a, b\}^* \mid |w|_a < |w|_b\}$
 - (d) $L_4 = \{w \in \{a, b, c\}^* \mid (|w|_a = 0) \Rightarrow (|w|_b = 0)\}$