

TD1 - Introduction to Formal Languages

February 4, 2025

1. Infer and describe in plain English the corresponding languages:

- (a) $L_1 = \{c, ac, abc, bc, bac, baabaac, ababac, \dots\}$ over $\Sigma_1 = \{a, b, c\}$.

Solution: L_1 is the set of words ending with a c . Formally, $L_1 = \{wc \mid w \in \Sigma_1^*\}$. For the language with a **single** final c , $L_1 = \{wc \mid w \in \Sigma_1^* \wedge \nexists v \in \Sigma_1^* \text{ s.t. } w = vc\}$.

- (b) $L_2 = \{\varepsilon, a, ab, aabb, aaa, aaabb, aaaab, \dots\}$ over $\Sigma_2 = \{a, b\}$.

Solution: L_2 is the set of words composed of a 's followed by a lesser number of b 's. Formally, $L_2 = \{a^m b^n \mid m \geq n \geq 0\}$.

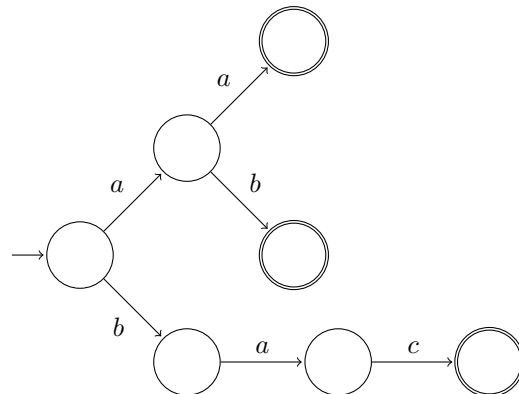
- (c) $L_3 = \{ab, abab, aaab, baab, bbbab, \dots\}$ over $\Sigma_3 = \{a, b\}$.

Solution: L_3 is the set of words with even length and ending with ab . Formally, $L_3 = \{wab \mid |w| \equiv 0 \pmod{2}\}$

2. Let $\Sigma = \{a, b, c\}$. Give finite-state automata that recognise the following languages:

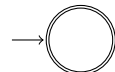
- (a) $L_1 = \{aa, ab, bac\}$.

Solution:



- (b) $L_2 = \{\varepsilon\}$.

Solution:



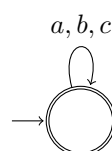
- (c) $L_3 = \emptyset$.

Solution:



- (d) $L_4 = \Sigma^*$.

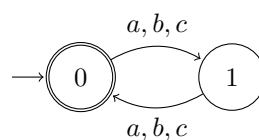
Solution:



3. Let $\Sigma = \{a, b, c\}$. Give finite state automata that accept the following languages:

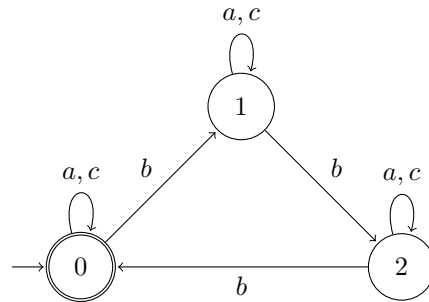
- (a) The set of words with an even length.

Solution:



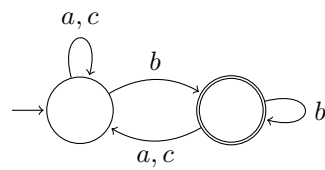
- (b) The set of words where the number of occurrences of b is divisible by 3.

Solution:



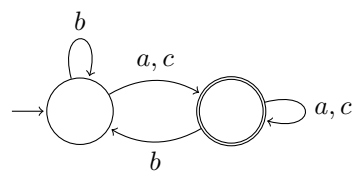
- (c) The set of words ending with a b .

Solution:



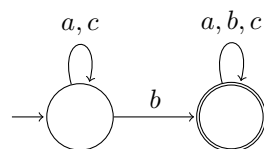
- (d) The set of non-empty words not ending with a b .

Solution:



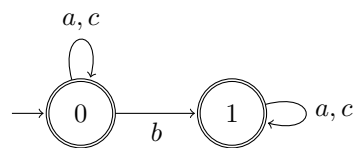
- (e) The set of words containing at least a b .

Solution:



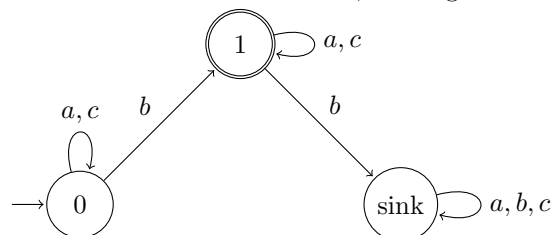
- (f) The set of words containing at most a b .

Solution:



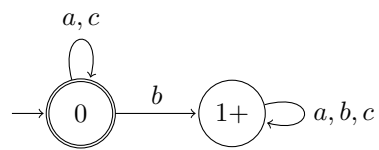
- (g) The set of words containing exactly one b .

Solution: I added a 'sink' state, although it was not necessary to include it.



- (h) The set of words not containing any b .

Solution:



- (i) The set of words with even length ending with ab .

Solution:

