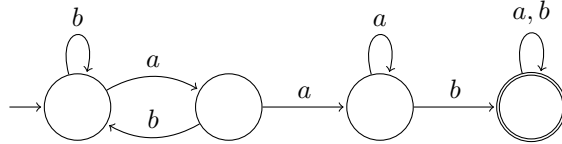


HW1 - Formal languages (Solution)

February 24, 2025

1. [4 pts.] Propose a complete deterministic finite state automaton that recognises the set of words over the alphabet $\{a, b\}$ containing the factors aab or $aaab$.

Solution:



2. [4 pts.] Let $\Sigma = \{a, b\}$. Let $L_1 = \{\varepsilon, aa, bb\}$ and $L_2 = \{a\}$ be two languages. Compute the following operations:

$$L_1^2 \quad L_2^* \quad L_1^2 \cap L_2^* \quad L_2^* \setminus L_1$$

Solution:

$$L_1^2 = \{\varepsilon, aa, bb, aaaa, aabb, bbaa, bbbb\}$$

$$L_2^* = \{a^n \mid n \geq 0\}$$

$$L_1^2 \cap L_2^* = \{\varepsilon, aa, aaaa\}$$

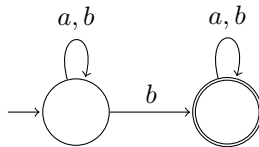
$$L_2^* \setminus L_1 = \{a^n \mid n \in \mathbb{N} \setminus \{0, 2\}\}$$

3. Let $\Sigma = \{a, b\}$. Propose finite state automata that recognise the underlying languages:

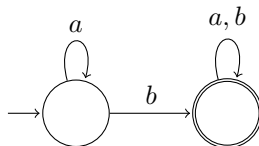
- (a) [1.5 pts.] $(a|b)^*b(a|b)^*$

Solution:

One solution is to simply translate each part of the regular expression separately. This yields a non-deterministic FSA:

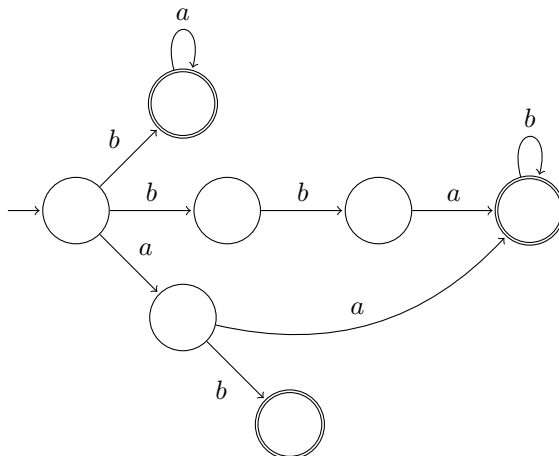


However, since the definition of the underlying language requires that **there be at least one b**, a simpler and deterministic FSA is given below:



- (b) [2.5 pts.] $ba^* + ab + (a + bb)ab^*$

Solution:



4. Let $\Sigma = \{a, b, c\}$. Give regular expressions that accept the following languages:

(a) [2 pts.] The set of words with even length ending with ab .

Solution: $((a + b + c)(a + b + c))^*ab$ or $((a + b + c)^2)^*ab$

(b) [2 pts.] The set of words containing at least three letters and whose third-to-last letter is an a or a c .

Solution: $(a + b + c)^*(a + c)(a + b + c)^2$

5. [4 pts.] Let $L = \{w \in \{a, b, c\}^* \mid (|w|_a = 0) \Rightarrow (|w|_b = 0)\}$. Is L rational? Justify.

Solution: Yes, $L = \mathcal{L}(c^* + (b + c)^*a(a + b + c)^*)$.