• **Syntax of λ-terms** Are the following λ-terms well formed ?

(1) a. $(x)x.x$
b. $\lambda x.\lambda y.\lambda z.u$
c. $\lambda y.(\lambda x.(y))x$
d. $\lambda x.(xx)$
e. $(x)\lambda y.x$

• **Syntax of λ-terms** Represent the following terms as (syntactic) trees.

(2) a. $\lambda f.\lambda g.\lambda x.(f)(g)x$
b. $\lambda f.\lambda g.\lambda x.((f)g)x$
c. $\lambda f.((\lambda g.\lambda x.f)g)x$

• **β-reduction :** Reduce as much as possible the following λ-terms

(3) a. $(\lambda x.(x)x)\lambda x.x$
b. $((\lambda x.\lambda y.(y)x)f)\lambda x.x$
c. $(\lambda n.\lambda f.\lambda x.(f)((n)f)x)\lambda(f)x.(f)x$

• **Redex & β-reduction** Identify all redexes in the following term, and reduce it as much as possible.

(4) $((\lambda S.\lambda V.(S)(V)\lambda Q.(Q)m)\lambda P.(P)j)\lambda O.\lambda y.(O)\lambda z.((kiss)y)z$

• **Notation conventions**
Since dot + parenthesis notation can become rather heavy, the following conventions are often adopted :

$$\begin{aligned} \lambda x_1.\lambda x_2 \ldots \lambda x_n.t &= \lambda x_1 x_2 \ldots x_n.t \\ t(t_1)(t_2)\ldots(t_m) &= t\, t_1 t_2 \ldots t_m \end{aligned}$$

Example : $\lambda xy.xy$ is read $\lambda x.\lambda y.x(y)$.
Note : Under this convention, the notation abc is not ambiguous : it corresponds to the term $((a)b)c$, or $af(af(a,b),c)$. To express the (different) term $af(a, af(b,c))$ at least one pair of parenthesis has to be inserted $(a)bc$.
Propose fully parenthesized versions of the following terms. If they can be reduced, reduce them.

(5) a. $\lambda xz.xyz$
b. $(\lambda x.\lambda y.fxy)xy$
c. $(\lambda x.\lambda y.xyy)\lambda y.\lambda a.y$

• **Church's integers** Check that Church's combinators for integers are working as intened : compute 1+2, 0×2, Succ(3). Count the number of necessary β-reductions.

$\mathsf{0} =_{\text{def}} \lambda f.\lambda x.x$	$\mathsf{Succ} =_{\text{def}} \lambda n.\lambda f.\lambda x.(f)((n)f)x$
$\mathsf{1} =_{\text{def}} \lambda f.\lambda x.(f)x$	$+ \equiv \lambda m.\lambda n.\lambda f.\lambda x.((m)f)((n)f)x$
$\mathsf{n} =_{\text{def}} \lambda f.\lambda x.(f)(f)\ldots(f)x$, with n times f	$\times \equiv \lambda m.\lambda n.\lambda f.(m)(n)f$