

Predicate Logic

N° 1. Translate as precisely as possible the following sentences into predicate logic. Explain the interpretation of non logical constants when it is not obvious. In case of ambiguity, propose as many formulae as necessary.

- (1)
- a. All architects have built a bridge.
 - b. Sam is depressed when no one understands her.
 - c. A child is confident only if no adult lies to him.
 - d. There are only two solutions to every problem.
 - e. Every student who solves a problem will explain it.
 - f. Any company which disappoint some customers will disappear if the all complaint.
 - g. Either no student will find an answer or all of them will.
 - h. A doctor will be sent to anyone who refuses that someone loves her.

N° 2. Among the formulae given in (2), indicate those which are appropriate representations for (3). Provide an explanation for those which are not appropriate.

- (2)
- a. $\forall x \left((Px \wedge \exists y (Cy \wedge Rxy)) \rightarrow Sxy \right)$
 - b. $\forall x \forall y \left((Px \wedge Cy \wedge Rxy) \rightarrow Sxy \right)$
 - c. $\forall x \left(Px \rightarrow \forall y \left((Cy \wedge Rxy) \rightarrow Sxy \right) \right)$
 - d. $\forall x \exists y \left((Px \wedge Cy \wedge Rxy) \rightarrow Sxy \right)$
- (3) Any person who raises a child is sacrificing for them.
Convention: $Sxy = x$ is sacrificing for y .

N° 3. Let us consider the following syllogism:

- (4)
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| <p style="margin: 0;">If a student fails, they didn't work correctly</p> <p style="margin: 0;">Alex is a student</p> <p style="margin: 0;">Alex failed</p> <hr style="border: 0.5px solid black;"/> <p style="margin: 0;">∴ Alex didn't work correctly</p> |
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1. Translate this syllogism into predicate logic.
2. Give the simplest possible model in which all the propositions are true.
3. A model-theoretic proof for this syllogism would try to prove that every model that makes the premisses true would also satisfy the conclusion. Sketch informally a proof of this sort.