

Exercise 1

Translate, as precisely as possible, the following sentences into propositional logic. Indicate to which sentence corresponds each propositional variable.

- (1) a. The engine is not noisy, but it uses lots of gas.
 b. It is not the case that Max comes if Pam or Sam comes.
 c. John is not only stupid, he is also mean.
 d. I go to the beach or to the movies by foot or by car.
 e. John will come only if Paul doesn't come.

Exercise 2

Show that for any φ , ψ and χ , the following pairs of formulae are logically equivalent :

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|------|-------------------------------------|--|-----------------|
| (1) | $\neg\neg\varphi$ | φ | |
| (2) | $\varphi \rightarrow \psi$ | $\neg\varphi \vee \psi$ | |
| (2') | $\varphi \rightarrow \psi$ | $\neg(\varphi \wedge \neg\psi)$ | |
| (3) | $\varphi \rightarrow \psi$ | $\neg\psi \rightarrow \neg\varphi$ | contraposition |
| (4) | $\varphi \leftrightarrow \psi$ | $(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$ | |
| (5) | $\varphi \leftrightarrow \psi$ | $(\varphi \wedge \psi) \vee (\neg\varphi \wedge \neg\psi)$ | |
| (6) | $\varphi \vee \varphi$ | φ | idempotence |
| (7) | $\varphi \wedge \varphi$ | φ | " |
| (8) | $\varphi \vee \psi$ | $\psi \vee \varphi$ | commutativity |
| (9) | $\varphi \wedge \psi$ | $\psi \wedge \varphi$ | " |
| (10) | $\varphi \vee (\psi \vee \chi)$ | $(\varphi \vee \psi) \vee \chi$ | associativity |
| (11) | $\varphi \wedge (\psi \wedge \chi)$ | $(\varphi \wedge \psi) \wedge \chi$ | " |
| (12) | $\varphi \wedge (\psi \vee \chi)$ | $(\varphi \wedge \psi) \vee (\varphi \wedge \chi)$ | distributivity |
| (13) | $\varphi \vee (\psi \wedge \chi)$ | $(\varphi \vee \psi) \wedge (\varphi \vee \chi)$ | " |
| (14) | $\neg(\varphi \wedge \psi)$ | $\neg\varphi \vee \neg\psi$ | de Morgan's law |
| (15) | $\neg(\varphi \vee \psi)$ | $\neg\varphi \wedge \neg\psi$ | " |

Exercise 3

Among the following discourses, which ones correspond to valid deductions ?

- (2) a. If Peter lied, then John is guilty. Yet John is guilty. Therefore Peter didn't lie.
 b. If Peter lied, then John is guilty. But Peter didn't lie. Therefore John isn't guilty.
 c. If Horacio loves Juliette, she will marry him. If Horacio doesn't love Juliette, she'll marry Gandalf. Yet Juliette will not marry Horacio, therefore she will marry Gandalf.
 d. If Horacio loves Juliette, she will marry him. If Horacio doesn't love Juliette, she'll marry Gandalf. Yet Juliette will marry Gandalf, therefore she won't marry Horacio.