Ex. 1

Let g be the grammar $S \to Sa$
$S \to a$

Describe informally the language of this grammar.

.... Answer ....

This is a regular grammar that generates all the words comprising one a and possibly any number of additional as. A rational expression would be aa^* or a^+.

Ex. 2

Propose a grammar that generates words composed of any number of as followed by exactly one b.

.... Answer ....

$S \to Xb$
$X \to Xa$
$X \to a$

Ex. 3

Remember that Dyck language is engendered by the grammar $S \to (S)$
$S \to \varepsilon$

Give a grammar such that evry word has exactly two ending parenthesis for every opening parenthesis, while remaining well balanced.

.... Answer ....

$S \to (S))$
$S \to \varepsilon$

Ex. 4

Give a grammar such that evry word has as many opening parenthesis than closing parenthesis.

.... Answer ....

$S \to (S)S$
$S \to)S(S$
$S \to \varepsilon$

Ex. 5

Modify the grammar $E \to E + E$; $E \to 1 \mid 2 \mid 3$ in such a way that ≪ (2+3)+1 ≫ is part of its language.

. . . . Answer . . . .

$E \to E + E$; $E \to 1 \mid 2 \mid 3$; $E \to (E)$

Ex. 6

Show that the grammar $E \to E + E$; $E \to E \times E$; $E \to 1 \mid 2 \mid 3$ is ambiguous.

. . . . Answer . . . .

It's enough to show that one specific word, e.g., $1 + 2 \times 3$ has two different derivation trees.

Ex. 7

Show that the grammar $E \to E + T \mid T, T \to T \times F \mid F, F \to (E) \mid a$ is **not** ambiguous.

Ex. 8

Donner une grammaire algébrique qui reconnaisse chacun des langages suivants (alphabet $X = \{a, b, c\}$).

- $L_0 = \{w \in X^* \ / \ w = a^n \ ; \ n \geq 0\}$
- $L'_0 = \{w \in X^* \ / \ w = a^n b^n ca \ ; \ n \geq 0\}$
- $L_1 = \{w \in X^* \ / \ w = a^n b^n c^p ; n > 0 \text{ et } p > 0\}$
- $L_2 = \{w \in X^* \ / \ w = a^n b^n a^m b^m ; n, m \geq 1\}$
- $L'_3 = \{w \in X^* \ / \ |w|_a = |w|_b\}$
- $L_3 = \{w \in X^* \ / \ |w|_a = 2|w|_b\}$
- $L_4 = \{w \in X^* \ / \ \exists x \in X^* \text{ tq } w = x\overline{x}\}$
- $L_5 = \{w \in X^* \ / \ w = \overline{w}\}$

.... Answer ....

- $L_0 = \{w \in X^* \mid w = a^n \; ; \; n \geq 0\}$
  $S \rightarrow aS|\varepsilon$
- $L'_0 = \{w \in X^* \mid w = a^n b^n ca \; ; \; n \geq 0\}$
  $S \rightarrow aSbX|\varepsilon \; ; \; X \rightarrow ca$
- $L_1 = \{w \in X^* \mid w = a^n b^n c^p ; n > 0 \text{ et } p > 0\}$
  La forme la plus simple : on charge S_1 de produire $a^n b^n$, et S_2 de produire c^p. D'autres formes sont possibles.

$$\begin{array}{lcl} S & \rightarrow & S_1 S_2 \\ S_1 & \rightarrow & aS_1b \mid ab \\ S_2 & \rightarrow & cS_2 \mid c \end{array}$$

- $L_2 = \{w \in X^* \mid w = a^n b^n a^m b^m ; n, m \geq 1\}$
- $L'_3 = \{w \in X^* \mid |w|_a = |w|_b\}$
- $L_3 = \{w \in X^* \mid |w|_a = 2|w|_b\}$
- $L_4 = \{w \in X^* \mid \exists x \in X^* \text{ tq } w = x\overline{x}\}$

- $L_5 = \{w \in X^* \mid w = \overline{w}\}$
  $= L_4 \cup X$

Ex. 9

Soient les deux grammaires suivantes. Pour chacune d'entre elles, donnez le langage engendré, et indiquez le type de la grammaire dans la classification de Chomsky. Commentez brièvement.

$$\begin{array}{lcl} S & \rightarrow & S_1 S_2 \\ S_1 & \rightarrow & aS_1b \mid ab \\ S_2 & \rightarrow & cS_2 \mid c \end{array}$$

$$\begin{array}{lcl} S & \rightarrow & aSBC \\ S & \rightarrow & aBC \\ CB & \rightarrow & BC \\ aB & \rightarrow & ab \\ bB & \rightarrow & bb \\ bC & \rightarrow & bc \\ cC & \rightarrow & cc \end{array}$$

.... Answer ....

Première grammaire : $a^n b^n c^p$; Deuxième grammaire : $a^n b^n c^n$. Une grammaire algébrique ne permet pas d'engendrer ce langage.