Formal Languages applied to Linguistics

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Overview

1. Formal Languages
2. Formal Grammars
3. Regular Languages
   - Definition
   - Automata
   - Properties
4. Formal complexity of Natural Languages

Pumping lemma: Intuition

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then some words have more than k letters.
Therefore, at least one state has to be "gone through" several times.
That means there is a loop on that state.
Then making any number of loops will end up with a word in L.

$\Rightarrow$ Pumping lemma

Pumping lemma: definition

Def. 18 (Pumping Lemma)

Let L be an infinite regular language.
There exists an integer k such that:

$$\forall x \in L,\ |x| > k,\ \exists u, v, w \text{ such that } x = uvw, \text{ with:}$$
$$\begin{array}{rl} (i) & |v| \geq 1 \\ (ii) & |uv| \leq k \\ (iii) & \forall i \geq 0,\ uv^i w \in L \end{array}$$

Pumping lemma: Illustration

Let's illustrate the lemma with a language which trivialy satisfies it: $a^* bc$.

Let $k = 3$, the work abc is long enough, and can be decomposed:

$\underbrace{\varepsilon}_{u} \quad \underbrace{a}_{v} \quad \underbrace{b \quad c}_{w}$

The three properties of the lemma are satisfied:

- $|v| \geq 1$ $(v = a)$
- $|uv| \leq k$ $(uv = a)$
- $\forall i \in \mathbb{N}$, $uv^i w (= a^i bc)$ belongs to the language by definition.

Pumping lemma: Consequences

The pumping lemma is a tool to prove that a language is **not** regular.

$\mathcal{L}$ regular	$\Rightarrow$	pumping lemma $(\forall i, uv^iw \in \mathcal{L})$
pumping lemma	$\not\Rightarrow$	$\mathcal{L}$ regular

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pumping lemma	$\not\Rightarrow$	$\mathcal{L}$ regular

to prove that $\mathcal{L}$ is

- regular: provide an automaton
- not regular: show that the pumping lemma does not apply

Pumping lemma: Consequences

Def. 19 (Consequences)

Let $\mathcal{A}$ be a k state automaton:

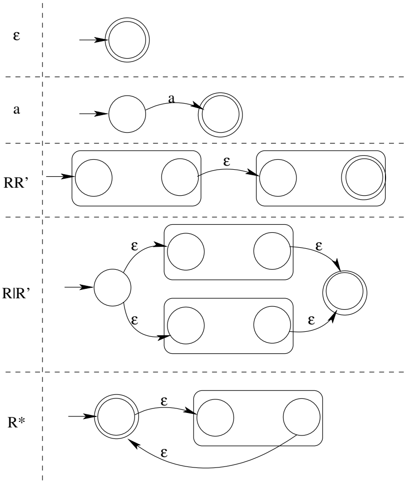
1. $L(\mathcal{A}) \neq \emptyset$ ***iff*** $\mathcal{A}$ recognises (at least) one word u s.t. $|u| < k$.
2. $L(\mathcal{A})$ is infinite ***iff*** $\mathcal{A}$ recognises (at least) one word u t.q. $k \leq |u| < 2k$.

Closure

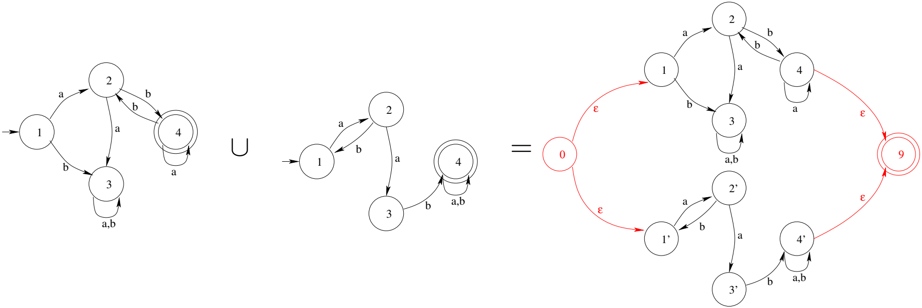
Regular languages are closed under various operations: if the languages L and L' are regular, so are:

- $L \cup L'$ (union); $L.L'$ (product); L^* (Kleene star)
  (rational operations)
- $L \cap L'$ (intersection); $\overline{L}$ (complement)
- ...

Rational operations



Union of regular languages: an example



Intersection of regular languages

Algorithmic proof
Deterministic complete automata

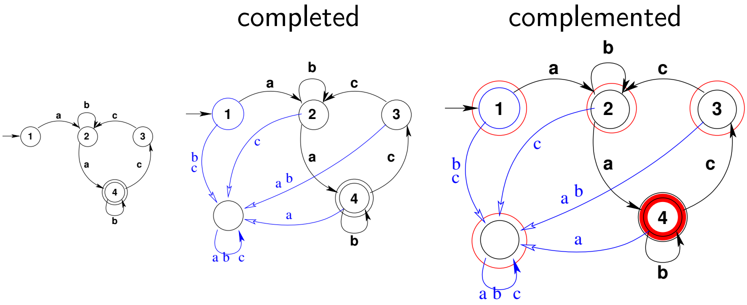
L_1	a	b
$\rightarrow$ 1	2	4
2	4	3
$\leftarrow$ 3	3	3
4	4	4

L_2	a	b
$\leftrightarrow$ 1	2	5
2	5	3
3	4	5
4	1	4
5	5	5

$L_1 \cap L_2$	a	b
$\rightarrow$ (1,1)	(2,2)	(4,5)
(2,2)	(4,5)	(3,3)
(4,5)	(4,5)	(4,5)
(3,3)	(3,4)	(3,5)
(3,4)	(3,1)	(3,4)
$\leftarrow$ (3,1)	(3,2)	(3,4)
(3,2)	(3,4)	(3,3)
(3,5)	(3,5)	(3,5)

Complement of a regular language

Deterministic complete automata



Results: expressivity

- Any finite langage is regular
- $a^n b^m$ is regular
- $a^n b^n$ is not regular
- ww^R is not regular (^{R} : reverse word)

Decidable problems

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- The "equivalence problem" $L(\mathcal{A}) \overset{?}{=} L(\mathcal{A}')$ is decidable.

$\Rightarrow$ it boils down to answering the question:

$$\left(L(\mathcal{A}) \cap \overline{L(\mathcal{A}')}\right) \cup \left(L(\mathcal{A}') \cap \overline{L(\mathcal{A})}\right) = \emptyset$$

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Motivation

Why an inquiry into the formal complexity of Natural Language(s) ?

- It gives us knowledge about the **structure** of natural languages,
- It helps us assess the **adequation** of linguistic formalisms,
- It gives bound for the **complexity** of NLP tasks,
- It provides us with **predictions** about human language processing.

Hypotheses

We assume that:

- We can talk about "natural language" in general: all languages have a similar structure, a similar power
- Natural languages are recursively enumerable, i.e. they are formal languages
- Natural languages are infinite

$\Rightarrow$ Under these hypotheses, it is possible to ask the question: what is the complexity of natural languages ?

An infinite number of sentences

1. Arbitrary long sentences can be built by adding new material:

(4) A stranger arrived.

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   (5) The cats hunt.

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An infinite number of sentences (cont'd)

Consider the 3 structures:

- If S_1, then S_2.
- Either S_1 or S_2.
- The man who said S_1 is coming today.

1. The colored items are *dependent* one from the other
2. It is possible to create nested sentences of arbitrary length:

(6) If either the man who said S_a is coming today, or S_b, then S_c.

$\Rightarrow$ A look at various ways to form infinite sentences gives access to complexity.

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Preliminaries: a word on lexicon

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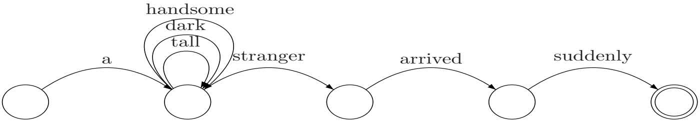

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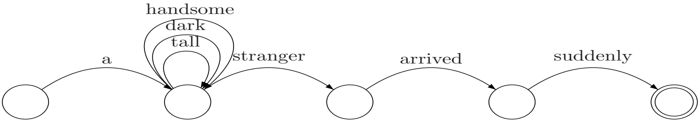

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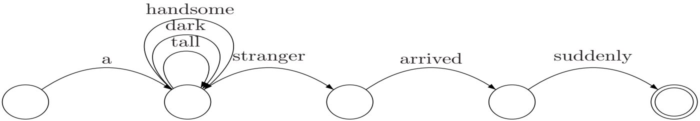

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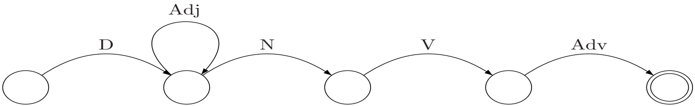

Chomsky's first attempt

Consider the 3 structures:

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2. It is possible to create nested sentences of arbitrary length:

(8) If either the man who said S_a is coming today, or S_b, then S_c.

Since such sentences are instances of mirroring and since the mirror language is not regular, then English is not regular (Chomsky, 1957, p. 22).

Fallacious claim: **a regular language may contain a non regular sub-language**

Classical argument I

Let's consider the sentence(s):

(9) A man fired another man.

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The sentences (10) are all well-formed sentences (for any n).

(10) A man (that a man)^n (hired)^n fired another man.

Classical Argument II

Let x = that a man
y = hired
w = a man
v = fired another man

- wx^*y^*v is regular
- English $\cap\ wx^*y^*v = wx^ny^nv$ (10)
- If English is regular, then wx^ny^nv must be regular (for the intersection of two regular languages is regular)
- But wx^ny^nv is not regular (pumping lemma).
  Contradiction $\Rightarrow$ English is not regular.

(Schieber, 1985)

Discussion

Counter arguments :

- Natural languages are finite
  - productivity doesn't seem to be bound
  - a list of all possible sentences, supposedly finite, is still too long for a human to learn
- People are bad at interpreting embedding: there might be a limit
  - there are indeed constraints on performance,
  - but in writing, or with an appropriate intonation, there doesn't seem to be a hard-wired limit

Discussion: processing problems with nested structures

Psycholinguistic evidence that (11b) is more accepted than (11a) (Fodor, Frazier)

(11) a. The patient who the nurse who the clinic had hired admitted met Jack.
b. The patient who the nurse who the clinic had hired met Jack.

Other factors:

(12) a. The pictures which the photographer who I met yesterday took were damaged by the child.
b. ?The pictures which the photographer who John met yesterday took were damaged by the child.

(13) a. Isn't it true that example sentences [that people [that you know] produce] are more likely to be accepted? (De Roeck et al, 1982)
b. A book [that some Italian [I've never heard of] wrote] will be published soon by MIT Press (Frank, 1992)

(Gibson & Thomas, 1997)

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Pumping lemma: intuition

1. If a word is long enough, then there is (at least) one non terminal symbol appearing several times in its derivation.

"long enough" ?

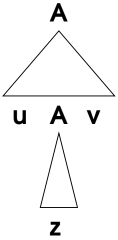
$$
\begin{array}{lcl}
S & \rightarrow & A\ B \\
A & \rightarrow & abaccabca \\
 & | & abSba \\
B & \rightarrow & ccccc
\end{array}
$$

Minimal length : 14:

$$S \rightarrow AB \rightarrow abaccabcaB \rightarrow abaccabcaccccc$$

Pumping lemma: intuition

2 Let's call this non terminal symbol A.



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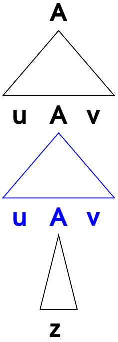

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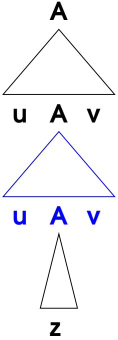

$$A \xrightarrow{*} uAv$$

$$A \xrightarrow{*} uAv \xrightarrow{*} uzv$$

$$A \xrightarrow{*} uAv \xrightarrow{*} uuAvv \xrightarrow{*} \underbrace{u \ldots u}_{n}\, z\, \underbrace{v \ldots v}_{n}$$

Pumping Lemma for CF languages

Def. 20 (Star lemma – CF languages)

If L is context-free, there exists $p \in \mathbb{N}$ such that:
$\forall w$ s.t. $|w| \geqslant p$,
w can be factorized $w = rstuv$,
with: $|su| \geqslant 1$
$|stu| \leqslant p$
$\forall i \geqslant 0, \quad rs^i tu^i v \in L$

(Bar-Hillel *et al.* , 1961)

Pumping lemma: Consequences

The pumping lemma gives us a tool to prove that a language is **not** context-free.

$$\boxed{\begin{array}{lcl} \mathcal{L} \text{ context-free} & \Rightarrow & \text{pumping lemma } (\forall i, rs^i tu^i v \in \mathcal{L}) \\ \text{pumping lemma} & \not\Rightarrow & \mathcal{L} \text{ context-free} \end{array}}$$

to prove that $\mathcal{L}$ is

context-free provide a type 2 grammar

not context-free show that the pumping lemma does not apply

Results: expressivity

- well-parenthetized words (dyck's language) is context-free
  $S \rightarrow (S)S \mid \varepsilon$
- $a^n b^n (n \geqslant 0)$ is a context-free language
  $S \rightarrow aSb \mid \varepsilon$
- $ww^R, w \in \Sigma^*$ (mirror language) is a context-free language
  $S \rightarrow aSa \mid bSb \mid \varepsilon$
- $ww, w \in \Sigma^*$ (copy language) is not context-free
  proof: pumping lemma
- $a^n b^n c^n$ is not context-free
  proof: pumping lemma
- $a^m b^n c^m d^n$ is not context-free
  proof: pumping lemma
- $xa^m b^n yc^m d^n z$ is not context-free
  proof: pumping lemma

Closure properties I

- CF languages are closed under rational operations
- union (gather all the rules, avoiding name conflicts, and adding a new start rule $S \to S_1 | S_2$),
- product ($S \to S_1 S_2$),
- and Kleene star ($S \to S_1 S \mid \varepsilon$).

Closure properties II : intersection

- CF languages are not closed under intersection

Example

$L_1 = \{a^i b^i c^j \mid i, j \geq 0\}$ is context-free:
$$S \to XY$$
$$X \to aXb \mid \varepsilon$$
$$Y \to cY \mid \varepsilon$$

$L_2 = \{a^i b^j c^j \mid i, j \geq 0\}$ is also context-free:
$$S \to XY$$
$$X \to aX \mid \varepsilon$$
$$Y \to bYc \mid \varepsilon$$

But $L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$ is not contex-free.

Closure properties III: other results

- CF languages are not closed under complement (since they are not closed under intersection)
- CF languages are closed under intersection with a regular language
- a sub-class of CF languages, *deterministic CF languages* are closed for set complement, but not for union (one can easily define an intrinsequely non deterministic language as the union of two "independant" languages)

Final argument I

After many attempts by various scholars, attempts which are severely critized and ruined in (Gazdar & Pullum, 1985), Schieber (1985) came up with a widely accepted answer:

1. In swiss-german, subordinate clauses can have a structure where all NPs precede all Vs. (14)

(14) Jan säit das mer NP* es huus haend wele V* aastrüche
Jan said that we NP* the house have wanted V* paint
'Jan said that we have wanted (that) V* NP* paint the house'

2. Among those subordinate clauses, those where all the dative NPs precede all the accusative NPs are well-formed. (15)

(15) ... das mer d'chind em Hans es huus haend wele laa hälfe aastrüche
... that we the_children.ACC Hans.DAT the house.ACC have wanted let help paint
'... that we have wanted to let the children help Hans to paint the house'

Final argument II

3. The number of verbs requiring a dative has to be equal to the number of dative NPs, the same for accusative.
4. The number of verbs in a subordinate clause is limited only by performance

Let R be the language:

R = {Jan säit das mer (d'chind)^h (em Hans)^i es huus haend wele (laa)^j (hälfe)^k aastrüche, $i, j, k, h \geqslant 1$}

Then let $L =$ Swiss-German $\cap\ R =$

{Jan säit das mer (d'chind)^m (em Hans)^n es huus haend wele (laa)^m (hälfe)^n aastrüche, $m, n \geqslant 1$}

L is not context-free, whereas R is regular.

$\Rightarrow$ Swiss-German is not context-free.

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Current proposal

1. The context-sensitive class seems too big: for instance $\{a^{2^i} \ / \ i \geqslant 0\}$ is context-sensitive.
2. Joshi (1985) proposed a subclass of type 1 languages, namely the class of *mildly context-sensitive languages* (MCSL), this class has the following properties:
   - ww is MCS
   - $a^n b^n c^n$ is MCS
   - $a^n b^n c^n d^n$ is MCS
   - $a^i b^j c^i d^j$ is MCS
   - $a^n b^n c^n d^n e^n$ is not MCS
   - www is not MCS
   - $ab^h ab^i ab^j ab^k ab^l, h > i > j > k > l \geqslant 1$ is not MCS
   - a^{2^i} is not MCS

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Conjecture : NL $\in$ MCSL

More about MCSL

Interesting properties of MCSL:

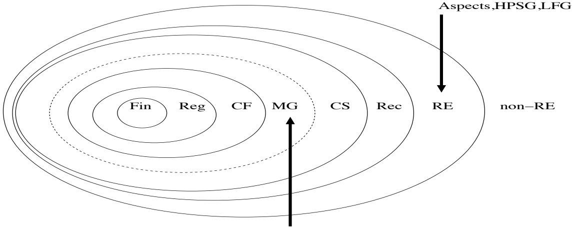
- restricted growth: if L is MCS, there is k such that for all words $w \in L$, there is a word w' s.t. $|w'| \leqslant |w| + k$
- word problem for MCSL are of a polynomial complexity

These properties are arguably common with natural languages

The formalism introduced by Joshi, *Tree Adjoining Grammars*, defines the class of MCSL.

Minimalist grammars (Stabler, 2011)

Minimalist grammars (MGs), as defined here by (5), (6) and (8), have been studied rather carefully. It has been demonstrated that the class of languages definable by minimalist grammars is exactly the class definable by multiple context free grammars (MCFGs), linear context free rewrite systems (LCFRSs), and other formalisms [62,64,66,41]. MGs contrast in this respect with some other much more powerful grammatical formalisms (notably, the 'Aspects' grammar studied by Peters and Ritchie [76], and HPSG and LFG [5,46,101]):



The MG definable languages include all the finite (Fin), regular (Reg), and context free languages (CF), and are properly included in the context sensitive (CS), recursive (Rec), and recursively enumerable languages (RE). Languages definable by tree adjoining grammar (TAG) and by a certain categorial combinatory grammar (CCG) were shown by Vijay Shanker and Weir to be sandwiched inside the MG class [103].[4] With all these results,

Theorem 1. $CF \subset \boxed{TAG \equiv CCG} \subset \boxed{MCFG \equiv LCFRS \equiv MG} \subset CS.$

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