2.8.1 Equivalences

• Bound variables are "dummy": their name no longer matters.

$$\forall x\, Fx \equiv \forall y\, Fy$$

But beware of unintended captures:

$$\forall x\,(Fx \wedge Gy) \not\equiv \forall y\,(Fy \wedge Gy)$$

• Duality rules (*de Morgan* laws)

$$\begin{aligned}
\forall x\ \alpha &\equiv \neg\exists\,\neg\alpha \\
&\textit{for instance:} \\
\forall x\ Rx &\equiv \neg\exists\,\neg Rx \\
\textit{All is relative} &\approx \textit{Nothing is absolute (}\approx \textit{non relative)} \\
\forall x\,(Px \to Kx) &\equiv \neg\exists x\,(Px \wedge \neg Kx) \\
\textit{All professors are kind} &\approx \textit{There are no non-kind professors} \\
&\textit{Other variants:} \\
\exists x\ \alpha &\equiv \neg\forall x\,\neg\alpha \\
\neg\exists x\ \alpha &\equiv \forall x\,\neg\alpha \\
\neg\forall x\ \alpha &\equiv \exists x\,\neg\alpha
\end{aligned}$$

• Distribution rules:

$$\begin{aligned}
\forall x\,(\alpha \wedge \beta) &\equiv (\forall x\ \alpha \wedge \forall x\ \beta) \\
\textit{All is rare and expensive} &\approx \textit{All is rare and all is expensive} \\
&\textbf{\textit{But:}} \\
\forall x\,(\alpha \vee \beta) &\not\equiv (\forall x\ \alpha \vee \forall x\ \beta) \\
\textit{All is either relative or absolute} &\not\approx \textit{Either all is relative or all is absolute}
\end{aligned}$$

$$\begin{aligned}
\exists x\,(\alpha \vee \beta) &\equiv (\exists x\ \alpha \vee \exists x\ \beta) \\
&\textbf{\textit{But:}} \\
\exists x\,(\alpha \wedge \beta) &\not\equiv (\exists x\ \alpha \wedge \exists x\ \beta)
\end{aligned}$$

$$\exists x\,(\alpha \to \beta) \equiv (\forall x\ \alpha \to \exists x\ \beta)$$

• Conditional distribution ($\bar{\beta}$ doesn't contain free occurrences of x)

$$\begin{aligned}
\bar{\beta} &\equiv \forall x\bar{\beta} \\
\bar{\beta} &\equiv \exists x\bar{\beta}
\end{aligned}$$

$$\begin{aligned}
\forall x\,(\alpha \vee \bar{\beta}) &\equiv (\forall x\ \alpha \vee \bar{\beta}) \\
\exists x\,(\alpha \wedge \bar{\beta}) &\equiv \exists x\ \alpha \wedge \bar{\beta} \\
\forall x\,(\alpha \to \bar{\beta}) &\equiv \exists x\ \alpha \to \bar{\beta} \\
\textit{Every entity is such that if it breaks, there is noise} &\approx \textit{If some entity breaks, there is noise} \\
\forall x\,(\bar{\beta} \to \alpha) &\equiv \bar{\beta} \to \forall x\ \alpha \\
\textit{For all person, if there is noise, s/he is upset} &\approx \textit{If there is noise, everyone is upset}
\end{aligned}$$