

## HW3 - First-order logic

April 6, 2025

1. For each of the following predicate logic formulae, number the (different) occurrences of the variables with subscripts according to their scope. For instance,

$$\exists x (\exists y Axyz \rightarrow Bxyz)$$

would become

$$\exists x_1 (\exists y_1 A x_1 y_1 z_1 \rightarrow B x_1 y_2 z_1).$$

- (a)  $\exists x (Axy \vee By)$
  - (b)  $\exists x (Axx \wedge \forall x Bxy) \vee \exists y Cy$
  - (c)  $\exists x (\exists y Axy \vee By)$
  - (d)  $\forall x \forall y ((Axy \wedge By) \rightarrow \exists w Cxw)$
  - (e)  $\forall x (\forall y Axx \rightarrow By)$
  - (f)  $\forall x \forall y Ayy \rightarrow Bx$
2. Simplify the following expression:

$$(q \rightarrow (p \wedge r)) \rightarrow (p \rightarrow q)$$

3. Translate as precisely as possible the following sentences into predicate logic. If ambiguous, provide a formula for each possible reading.

- (1)
    - a. All superheroes had a difficult time.
    - b. Every time Mia finds a wallet she gives it back to its owner.
    - c. All newspapers which don't have readers will disappear if they don't find a buyer.
    - d. Every student who solves a problem will explain it.
    - e. There are only two solutions to every problem.
    - f. Everyone is marked by an unrequited love.
    - g. Nobody who is not a French likes cheese better than any French.
    - h. Only completely consistent people are dead.
4. Consider a language that includes binary predicates  $P$  and equality ( $=$ ). Consider the three following formulae:

$$(F_1) \forall x \forall y (Pxy \rightarrow x \neq y) \quad (F_2) \forall x \forall y (Pxy \wedge x \neq y) \quad (F_3) \forall x \exists y (Pxy \wedge x \neq y)$$

For each formula, determine whether there exists a model  $\mathcal{M} = \langle D, I \rangle$  that satisfies it:

- (a) when  $D$  is a singleton,
  - (b) when  $D$  contains exactly two elements,
  - (c) when  $D = \mathbb{N} = \{0, 1, 2, \dots\}$  and  $I(P) = \{(x, y) \mid x \text{ is divisible by } y\}$ .
5. (a) Translate the following sentences into predicate logic.
- (2)
    - a. John owns everything he has not lost.
    - b. John has not lost 1 million dollars.
    - c. John owns one million dollars.
- (b) Analyse the syllogism going from the conjunction of (2a) and (2b) to the conclusion (2c). Explain where the problem lies.