HW2 - Formal grammars

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1. What are the languages engendered by the following context-free grammars:
   (a) $\mathcal{G}_1 = (\{a, b\}, \{S\}, S, \{S \to aSa \mid bSb \mid \varepsilon\})$
   (b) $\mathcal{G}_2 = (\{a, b\}, \{S, T\}, S, \{S \to aSa \mid bSb \mid aTb \mid bTa, T \to Ta \mid Tb \mid \varepsilon\})$

2. Give a context-free grammar for each of the following languages:
   (a) $\{a^m b^n c^p \mid n \geq m + p\}$
   (b) $\{va^n \mid v \in \{a, b\}^n \ \wedge \ |v|_a = n\}$
   (c) $\{w \in \{a, b\}^* \mid |w|_a \geq |w|_b\}$

3. Propose a context-free grammar that generates the set of valid arithmetic expressions (in $\mathbb{N}$) over $\{0, 1, \ldots, 9, \times, +, (,)\}$ with the two following restrictions:
   - the numbers should not contain any leading 0's, except for 0 itself, and
   - there should not be any redundant parentheses.

   For instance, expressions in (1) are valid, while those in (2) are not.

   (1) a. $1234 \times 0 \times (1 + 654 + 292929)$
   b. $4 \times 911 + 2 \times (42 + 5555 \times 886) \times 1$
   c. $9 \times (1 + 2 + 7) \times (4 \times 2 + 1) + 5 \times 112$
   d. $(0 + 4) \times (5 \times 7 + 91) + 4$

   (2) a. $^*\, 010$
   b. $^*\, 1 \times (2)$
   c. $^*\, 4 \times 911 + (2 \times 42 + 5555)$
   d. $^*\, 9 \times (1 \times 2) + 74$

   Is your grammar ambiguous? If so, show two parse trees for a valid expression; otherwise, explain informally why your grammar is not (a formal proof is not expected).

4. Define a non-trivial context-free grammar that could generate the sentences in (3)

   (3) a. Alice eats a sweet.
   b. The waiter gives Alice cakes in a tray.
   c. The tabby cat with a grin disappears.
   d. Alice likes that her brother plays football with his friends.